OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Use formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+1) = \tfrac{1}{3}n(n+1)(n+2).$$ [5]

2 The cubic equation $x^3 - 6x^2 + kx + 10 = 0$ has roots $p - q$, p and $p + q$, where q is positive.

(i) By considering the sum of the roots, find p. [2]

(ii) Hence, by considering the product of the roots, find q. [3]

(iii) Find the value of k. [3]

3 The complex number $2 + \mathrm{i}$ is denoted by z, and the complex conjugate of z is denoted by z^*.

(i) Express z^2 in the form $x + \mathrm{i}y$, where x and y are real, showing clearly how you obtain your answer. [2]

(ii) Show that $4z - z^2$ simplifies to a real number, and verify that this real number is equal to zz^*. [3]

(iii) Express $\frac{z+1}{z-1}$ in the form $x + \mathrm{i}y$, where x and y are real, showing clearly how you obtain your answer. [3]

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 3^{2n} - 1.$$

(i) Write down the value of u_1. [1]

(ii) Show that $u_{n+1} - u_n = 8 \times 3^{2n}$. [3]

(iii) Hence prove by induction that each term of the sequence is a multiple of 8. [4]

5 **(i)** Show that

$$\frac{1}{2r-1}-\frac{1}{2r+1}=\frac{2}{4r^2-1}.$$ [2]

(ii) Hence find an expression in terms of n for

$$\frac{2}{3}+\frac{2}{15}+\frac{2}{35}+\ldots+\frac{2}{4n^2-1}.$$ [4]

(iii) State the value of

(a) $\displaystyle\sum_{r=1}^{\infty}\frac{2}{4r^2-1}$, [1]

(b) $\displaystyle\sum_{r=n+1}^{\infty}\frac{2}{4r^2-1}$. [1]

6 In an Argand diagram, the variable point P represents the complex number $z=x+\mathrm{i}\,y$, and the fixed point A represents $a=4-3\mathrm{i}$.

(i) Sketch an Argand diagram showing the position of A, and find $|a|$ and $\arg a$. [4]

(ii) Given that $|z-a|=|a|$, sketch the locus of P on your Argand diagram. [3]

(iii) Hence write down the non-zero value of z corresponding to a point on the locus for which

(a) the real part of z is zero, [1]

(b) $\arg z=\arg a$. [2]

7 The matrix $\mathbf{A}$ is given by $\mathbf{A}=\begin{pmatrix}1 & -2\\ 2 & 1\end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by **A**. [3]

(ii) The value of $\det\mathbf{A}$ is 5. Show clearly how this value relates to your diagram in part **(i)**. [3]

A represents a sequence of two elementary geometrical transformations, one of which is a rotation R.

(iii) Determine the angle of R, and describe the other transformation fully. [3]

(iv) State the matrix that represents R, giving the elements in an exact form. [2]

8 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 2 & -1 \\ 2 & 3 & -1 \\ 2 & -1 & 1 \end{pmatrix}$, where a is a constant.

(i) Show that the determinant of **M** is $2a$. [2]

(ii) Given that $a \neq 0$, find the inverse matrix $\mathbf{M}^{-1}$. [4]

(iii) Hence or otherwise solve the simultaneous equations

$$\begin{aligned} x + 2y - z &= 1, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0. \end{aligned}$$

[3]

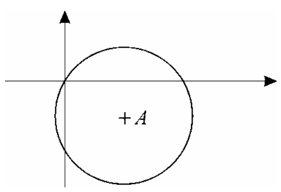
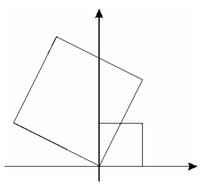
(iv) Find the value of k for which the simultaneous equations

$$\begin{aligned} 2y - z &= k, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0, \end{aligned}$$

have solutions. [3]

(v) Do the equations in part **(iv)**, with the value of k found, have a solution for which $x = z$? Justify your answer. [2]

1	$\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	M1	For considering the two separate sums
		A1	For either correct sum formula stated
		A1	For completely correct expression
	$= \frac{1}{6}n(n+1)(2n+1+3) = \frac{1}{3}n(n+1)(n+2)$	M1	For factorising attempt
		A1 **5**	For showing given answer correctly
		[5]	
2	**(i)** $(p-q)+p+(p+q)=6 \Rightarrow p=2$	M1	For use of $\Sigma\alpha = -b/a$
		A1 **2**	For correct answer
	(ii) $2(2-q)(2+q) = -10$	B1√	For use of $\alpha\beta\gamma = -d/a$
	Hence $4-q^2 = -5 \Rightarrow q = 3$	M1	For expanding and solving for q^2
		A1 **3**	For correct answer
	(iii) *EITHER*: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
	$-1\times2+2\times5+-1\times5=k$	M1	For use of $\Sigma\alpha\beta = c/a$
	i.e. $k=3$	A1√	For correct answer from their roots
	OR: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
	Equation is $(x+1)(x-2)(x-5)=0$	M1	For stating and expanding factorised form
	Hence $k=3$	A1√ **3**	For correct answer from their roots
		[8]	
3	**(i)** $z^2 = (2+\mathrm{i})^2 = 4+4\mathrm{i}+\mathrm{i}^2 = 3+4\mathrm{i}$	M1	For showing 3-term or 4-term expansion
		A1 **2**	For correct answer
	(ii) $4z - z^2 = 8+4\mathrm{i}-3-4\mathrm{i} = 5$	B1	For correct value 5
	$zz^* = (2+\mathrm{i})(2-\mathrm{i}) = 5$	B1	For stating or using $z^* = 2-\mathrm{i}$
		B1 **3**	For correct verification of given restult
	(iii) $\frac{z+1}{z-1} = \frac{3+\mathrm{i}}{1+\mathrm{i}} = \frac{(3+\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})} = \frac{4-2\mathrm{i}}{2} = 2-\mathrm{i}$	B1	For correct initial form $\frac{3+\mathrm{i}}{1+\mathrm{i}}$
		M1	For multiplying top and bottom by $1-\mathrm{i}$
		A1 **3**	For correct answer $2-\mathrm{i}$
		[8]	
4	**(i)** $u_1 = 8$	B1 **1**	For correct value stated
	(ii) $3^{2(n+1)} - 1 - (3^{2n}-1) = 9\times3^{2n} - 3^{2n} = 8\times3^{2n}$	B1	For stating or using $u_{n+1} = 3^{2(n+1)} - 1$
		M1	For relevant manipulation of indices in u_{n+1}
		A1 **3**	For showing given answer correctly
	(iii) u_1 is divisible by 8, from **(i)**	B1	For explicit check for u_1
	Suppose u_k is divisible by 8, i.e. $u_k = 8a$	M1	For induction hypothesis u_k is mult. of 8
	Then $u_{k+1} = u_k + 8\times3^{2k} = 8(a+3^{2k}) = 8b$	M1	For obtaining and simplifying expr. for u_{k+1}
	i.e. u_{k+1} is also divisible by 8, and result follows by the induction principle	A1 **4**	For correct conclusion, stated and justified
		[8]	

Q	Part	Answer	Mark	Total	Guidance
5	(i)	LHS $= \dfrac{2r+1-(2r-1)}{(2r-1)(2r+1)} = \dfrac{2}{4r^2-1} =$ RHS	M1		For correct process for adding fractions
			A1	2	For showing given result correctly
	(ii)	Sum is $\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$	M1		For expressing terms as differences using **(i)**
			A1		For at least first two and last terms correct
		This is $1-\dfrac{1}{2n+1}$	M1		For cancelling pairs of terms
			A1	4	For any correct form
	(iii)	**(a)** Sum to infinity is 1	B1$\surd$	1	For correct value; follow their **(ii)** if cnvgt
		(b) Required sum is $\dfrac{1}{2n+1}$	B1$\surd$	1	For correct difference of their **(iii)(a)** and **(ii)**
				8	
6	(i)	(See diagram in part **(ii)** below)	B1		For point *A* correctly located
		$\lvert a \rvert = \sqrt{(3^2+4^2)} = 5$	B1		For correct value for the modulus
		$\arg a = -\tan^{-1}\left(\frac{3}{4}\right) = -0.644$	M1		For any correct relevant trig statement
			A1	4	For correct answer (radians or degrees)
	(ii)		B1		For any indication that locus is a circle
			B1		For any indication that the centre is at *A*
			B1	3	For a completely correct diagram
	(iii)	**(a)** $z = -6\text{i}$	B1	1	For correct answer
		(b) $z = 8-6\text{i}$	M1		For identification of end of diameter thru *A*
			A1	2	For correct answer
				10	
7	(i)	$\begin{pmatrix}1 & -2\\ 2 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 0 & 1\\ 0 & 0 & 1 & 1\end{pmatrix}=\begin{pmatrix}0 & 1 & -2 & -1\\ 0 & 2 & 1 & 3\end{pmatrix}$	M1		For at least one correct image
			A1		For all vertices correct
			A1	3	For correct diagram
	(ii)	The area scale-factor is 5	B1		For identifying det as area scale factor
		The transformed square has side of length $\sqrt{5}$	M1		For calculation method relating to large sq.
		So its area is 5 times that of the unit square	A1	3	For a complete explanataion
	(iii)	Angle is $\tan^{-1}(2) = 63.4°$	B1		For $\tan^{-1}(2)$, or equivalent
		Enlargement with scale factor $\sqrt{5}$	B1		For stating 'enlargement'
			B1	3	For correct (exact) scale factor
	(iv)	$\begin{pmatrix}\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$	M1		For correct $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ pattern
			A1	2	For correct matrix in exact form
				11	

8	(i)	$\det \mathbf{M} = a(3-1) - 2(2-(-2)) - 1(-2-6)$	M1	For correct expansion process
		$= 2a$	A1 **2**	For showing given answer correctly
	(ii)	$\mathbf{M}^{-1} = \frac{1}{2a}\begin{pmatrix} 2 & -1 & 1 \\ -4 & a+2 & a-2 \\ -8 & a+4 & 3a-4 \end{pmatrix}$	M1	For correct process for adjoint entries
			A1	For at least 4 correct entries in adjoint
			B1	For dividing by the determinant
			A1 **4**	For completely correct inverse
	(iii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, with $a = 1$	B1	For correct statement involving inverse
		So $x = 0,\ y = 1,\ z = 1$	M1	For carrying out the correct multiplication
			A1 **3**	For all three correct values
	(iv)	Eliminating x gives $4y - 2z = 2$	M1	For eliminating x from 2nd and 3rd equns
		So for consistency with 1st equn, $k = 1$	M1	For comparing two y-z equations
			A1 **3**	For correct value for k
	(v)	Solving $x + 3y = 2,\ 3x - y = 0$ gives $x = \frac{1}{5},\ y = \frac{3}{5}$	M1	For using $x = z$ to solve a pair of equns
		These values check in $2y - x = 1$, so soln exists	A1 **2**	For a completely correct demonstration
			14	